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2003 J. Phys.: Condens. Matter 15 2693

(http://iopscience.iop.org/0953-8984/15/17/321)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **15** (2003) 2693–2700 PII: S0953-8984(03)55232-8

Integer quantum Hall transitions in the presence of off-diagonal disorder

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Received 25 October 2002, in final form 21 March 2003
Published 22 April 2003
Online at stacks.iop.org/JPhysCM/15/2693

Abstract

We study the disorder-induced quantum Hall-to-insulator transition in a lattice model with only off-diagonal disorder. The localization length of the system is calculated by using the finite-size scaling method combined with the transfer-matrix technique. By increasing the off-diagonal disorder strength we find that the extended states do not float up in energy, but move towards the direction of lower energy. As a consequence, a direct transition from a higher integer quantum Hall state to an insulator becomes possible, in agreement with previous studies based on a tight-binding model with only diagonal disorder.

1. Introduction

The scaling theory of localization [1] predicts that all one-particle states in a two-dimensional (2D) system are localized in the absence of magnetic field. In a strong perpendicular magnetic field, however, extended states appear in the centres of disorder-broadened Landau bands where the localization length of electronic states diverges [2]. This effect has been intensively studied in connection with the integer quantum Hall effect (IQHE).

Recently, there has been a great renewal of interest in studying the problem of the disorder-induced quantum Hall-to-insulator transition. Despite a lot of numerical and theoretical studies, the issue of how extended states at the centres of the Landau bands evolve with disorder strength or magnetic field remains controversial. Khmelnitskii [3] and Laughlin [4] both argued that the extended states do not disappear discontinuously or merge together but rather float up in energy in the limit of weak magnetic field or strong randomness. This levitation picture is crucial to the global phase diagram of the IQHE proposed by Kivelson *et al* [5], which predicts that at a quantum transition the Hall conductance σ_{xy} can only change by e^2/h as the filling factor n changes and the final transition to the insulator always happens from the lowest ($n = 1$) Hall plateau. But such a floating scenario for extended states has been challenged by recent numerical studies [6–8] based on a tight-binding model with diagonal disorder, in which the extended states are found to disappear without involving a floating-up process as disorder

strength increases. It has been argued [8] that this behaviour is closely related to the lattice effect and can be well understood in terms of annihilation of the topological Chern numbers. However, recent numerical calculations [9, 10] have shown that floating up of the extended states can also take place in the lattice model if long-range-correlated disorder potentials are considered.

Most of the previous numerical studies focused on the tight-binding model with only diagonal disorder. The role of off-diagonal randomness in the extended states of the IQHE has not been investigated up to now. Recent studies [11–14] showed that 2D systems with purely off-diagonal disorder might display anomalous localization behaviour due to a particle–hole or chiral symmetry. Therefore, it is of particular interest to investigate the influence of off-diagonal disorder on the extended states in the IQHE.

In the present paper, we study how the extended states in the IQHE evolve with off-diagonal disorder strength on the basis of a tight-binding model. We calculate the localization length of such systems by the finite-size scaling method combined with the transfer-matrix technique. In the case of weak disorder, a series of extended states are found at the centre of each Landau band. With increasing disorder strength, the positions of the extended states do not float up in energy but move towards the direction of lower energy. By studying the localization behaviour of the system at a fixed Fermi energy near the centre of the nth plateau at weak disorder, a direct transition to an insulator ($n \to 0$) is revealed as disorder strength W is increased. The critical disorder W_c for the $n \to 0$ transition is found to monotonically decrease with the decrease of the magnetic field, and can be extrapolated to zero at zero magnetic field.

The paper is organized as follows: in the next section we present the model and the basic formalism; in section 3 the numerical results are illustrated; and the last section is devoted to a brief summary and discussion.

2. Model and formulism

We consider a tight-binding model on a square lattice with random nearest-neighbour hoppings and a uniform magnetic field described by the Hamiltonian

$$H = \sum_{\langle ij \rangle} (t_{ij} \mathrm{e}^{\mathrm{i}\alpha_{ij}} c_i^\dagger c_j + \mathrm{H.c.}) \tag{1}$$

where $c_m^\dagger$ is the fermionic creation operator on site i and the nearest-neighbour hopping integrals t_{ij} are real random variables satisfying the probability distribution

$$P(\ln t_{ij}) = \frac{1}{W}, \qquad \text{for } -\frac{W}{2} \leqslant \ln t_{ij}/t_0 \leqslant \frac{W}{2}. \tag{2}$$

This disorder guarantees positive hoppings $t_{i,j}$ with typical variation of the random element W that is believed to be a good measure for the strength of the off-diagonal disorder [14, 15]. In the following, we will set t_0 as the unit of energy ($t_0 = 1$). The effect of the magnetic field appears in the complex phases of the transfer terms which in the Landau gauge read

$$\alpha_{ij} = \begin{cases} \pm 2\pi\alpha(\boldsymbol{r}_i \cdot \boldsymbol{e}_x), & \text{if } \boldsymbol{r}_i = \boldsymbol{r}_j \pm \boldsymbol{e}_y, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where $\boldsymbol{r}_i$ denotes the position of site i with lattice constant $a = 1$, $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are unit vectors in the x- and y-directions, and α is the amount of magnetic flux per plaquette in units of the flux quantum hc/e. Here, we concentrate on the case $\alpha = 1/q$ (where q is an integer), so the tight-binding band splits into q subbands.

On a square lattice, site i can be labelled by its coordinates in the x- and y-directions, denoted by integers m and n, respectively. The corresponding Schrödinger equation can be written as

$$t_{m,n;m+1,n}\psi_{m+1,n} = E\psi_{m,n} - t_{m,n;m-1,n}\psi_{m-1,n} - t_{m,n;m,n-1}\mathrm{e}^{-\mathrm{i}\,2\pi m\alpha}\psi_{m,n-1} - t_{m,n;m,n+1}\mathrm{e}^{\mathrm{i}\,2\pi m\alpha}\psi_{m,n+1}. \tag{4}$$

where E is the energy of electrons and $\psi_{m,n}$ denotes the amplitude of the wavefunction at site (m, n).

In order to calculate the localization length of electrons in a finite square system with linear size M, we use the finite-size scaling method combined with the transfer-matrix technique. We calculate the damping of wavefunctions in the x-direction for a long strip of size $M \times L$ with L being extremely large. The periodic boundary condition is adopted in the y-direction. For a given energy E, a $2M \times 2M$ transfer matrix T_i can be easily set up, mapping the wavefunction amplitudes at column $m - 1$ and m to those at column $m + 1$ in the strip. The propagation along the strip is therefore described by the product of transfer matrices

$$Q_L = \prod_{i=1}^{L} T_i. \tag{5}$$

The transfer matrix of equation (5) has M pairs of eigenvalues whose logarithms correspond to the Lyapunov exponents of wavefunctions [16], $(\gamma_i, -\gamma_i)$ $\{i = 1, 2, \ldots, M\}$. The largest localization length $\lambda_M(E)$ for energy E in a system with finite width M is given by the inverse of the smallest Lyapunov exponent. In our numerical calculation, we choose the length of the strip L to be over 2×10^5 so that the self-averaging effect automatically takes care of the ensemble statistical fluctuation. We estimate and control the fluctuations from the deviations of the calculated eigenvalues of two adjacent iterations which show satisfactory suppression on increasing L.

We use the standard one-parameter finite-size scaling ansatz [16] to obtain the thermodynamic localization length ξ. According to the one-parameter scaling theory, the rescaled localization length λ_M/M can be expressed in terms of a universal function of M/ξ, i.e.,

$$\frac{\lambda_M(E, W)}{M} = f\left(\frac{M}{\xi(E, W)}\right), \tag{6}$$

where $f(x) \propto 1/x$ in the thermodynamic limit $M \to \infty$ for localized states while it approaches a constant ($\sim$1) when ξ diverges. Due to the symmetry of the lattice model, in what follows we only consider the negative energy region $E < 0$.

3. Numerical results and discussion

We first study the localization behaviour of the system in the case of weak disorder at a fixed magnetic field. As an example, figure 1(a) displays the rescaled localization length λ_M/M as a function of energy E with $\alpha = 1/8$ and $W = 0.4$ for different widths M of the system. In this case, four well-defined Landau bands can be clearly seen. At the centres of the Landau bands the rescaled localization length is independent of sample size M, which indicates that the localization length increases linearly with increasing M. Therefore, the peaks of λ_M/M denote the positions of the extended states in the thermodynamic limit. By contrast, λ_M/M for energies away from the Landau band centres decreases as M increases, and hence they are truly localized states. In figure 1(b), we plot the rescaled localization length λ_M/M as a function M for energies close to the lowest Landau band centre. In order to minimize the uncertainty in the

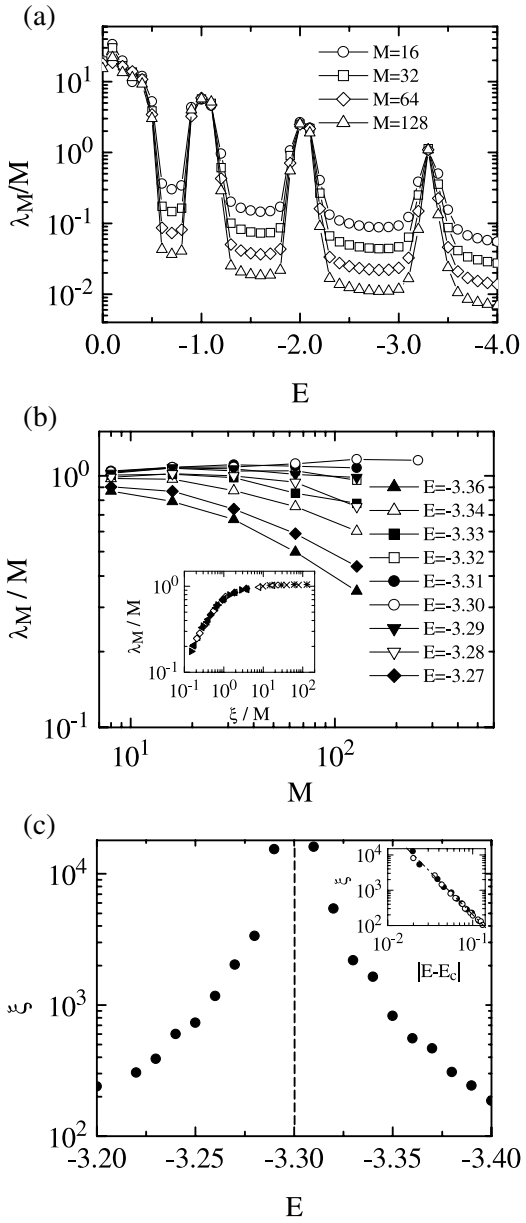

Figure 1. (a) The rescaled localization length λ_M/M as a function of energy E with $\alpha = 1/8$ and $W = 0.4$. (b) The rescaled localization length λ_M/M for the strip system calculated for $M = 8, 16, 32, 64, 128$, and 256 for energy close to the lowest Landau band. Inset: the one-parameter scaling function. (c) The divergence of the thermodynamic localization length ξ at the critical energy E_c. Inset: ξ versus $|E - E_c|$ in a log–log plot with $\nu = 2.34 \pm 0.02$.

calculation due to the finite-size effect, we adopt a strip of width up to $M = 256$ to obtain the critical energy E_c. With increasing energy from the lower side of the lowest subband centre, an M-independent λ_M/M can be clearly seen at a critical energy $E_c \approx -3.3$. But away from E_c, λ_M/M monotonically decreases as M increases, showing a feature of localized states. All

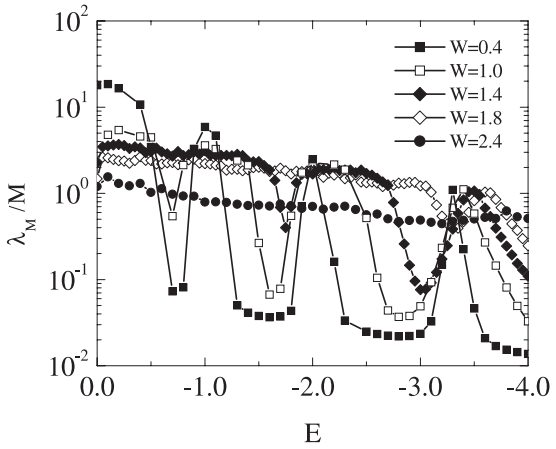

Figure 2. The rescaled localization length λ_M/M as a function of E for different disorder strengths with $\alpha = 1/8$ and $M = 64$.

the data in figure 1(b) can be collapsed onto a single curve $\lambda_M(E)/M = f[M/\xi(E)]$ as shown in the inset of figure 1(b), where ξ is the thermodynamic localization length. In figure 1(c), we show the thermodynamic localization length ξ as a function of energy E. One can clearly see that ξ diverges at E_c, which again confirms the existence of a delocalized state at the centre of the band. Our best-fit analysis gives a critical energy $E_c = -3.30$ with a critical exponent $\nu = 2.34 \pm 0.02$ appearing in $\xi = \xi_0|E - E_c|^{-\nu}$ on the two sides of E_c (the critical exponent ν is given by the slope of the straight line in the inset log–log plot of figure 1(c)). The localization length exponent ν is in remarkable agreement with the analytical value [2] $\nu \simeq \frac{7}{3}$ and previous numerical results [17–19] obtained for tight-binding models with only diagonal disorder.

To illustrate how the extended states evolve with the disorder strength, we have calculated the energy dependence of the rescaled localization length λ_M/M for a fixed system width $M = 64$ with the same field strength as in figure 1. As presented in figure 2, λ_M/M decreases with increasing strength of disorder. The peaks associated with higher-energy Landau bands are destroyed earlier than those of lower ones, and the peak for the lowest Landau band is the last to vanish at a disorder of about $W \approx 2.4$, beyond which no extended states exist in the whole energy regime. In the entire range of disorder strength shown in figure 2, the peak positions of the Landau bands do not float up in energy, but move down towards the low-energy regime. Thus, in contrast to the case of diagonal disorder, in which extended states are found to disappear without changing their positions at strong disorder, the displayed results clearly indicate a moving down of the extended states before they disappear.

Figure 2 also shows that the second-lowest extended state moves distinctly faster than the lowest one, so the first plateau between these two extended states should be narrowed during this 'float-down' process. If such a narrowing is to continue with the increase of the disorder strength, then it can eventually result in the vanishing of the first plateau, leading to a direct plateau–insulator transition. To investigate this possibility, we have calculated the disorder dependence of the rescaled localization length by fixing the Fermi energy near the centre of the $n = 2$ plateau ($E = -1.6$) at weak disorders. The results are shown in figure 3(a) where λ_M/M is displayed versus W for different system widths M ($M = 16$, 32, 64, and 128). With increasing sample width M, an M-independent fixed point appears at a critical disorder strength $W_c \approx 1.3$, indicating an extended state. But away from W_c, λ_M/M monotonically decreases with increase of M as a typical behaviour in localized states. Thus the existence of

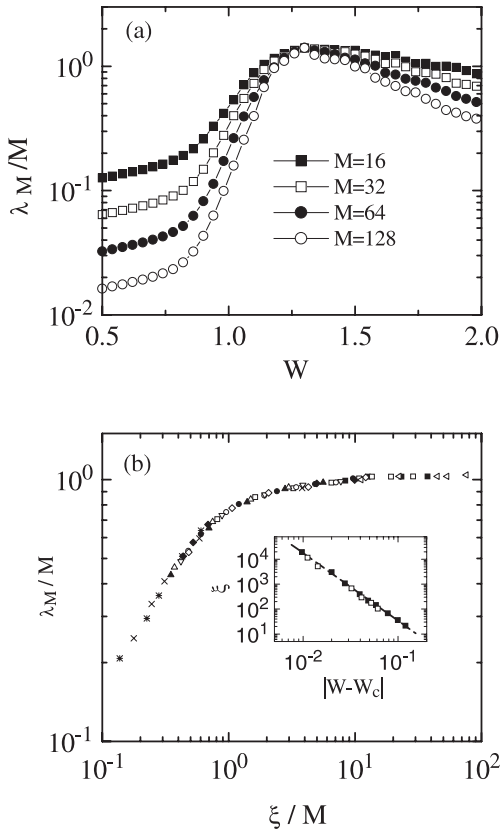

Figure 3. (a) The rescaled localization length λ_M/M as a function of disorder strength W for $E = -1.6$, which corresponds to the centre of the $n = 2$ IQHE plateau at weak disorder. (b) The scaling function at $E = -1.6$. Here, different symbols represent different disorder strengths W. Inset: ξ versus $|W - W_c|$ in a log–log plot with $\nu = 2.42 \pm 0.01$.

a single W_c indicates a direct $n = 2 \to 0$ transition, which is forbidden in the global phase diagram. The finite-size scaling method can also be applied to the $n = 2 \to 0$ transition to verify the one-parameter scaling theory. As shown in figure 3(b), by collapsing the data in figure 3(a) as $\lambda_M(W)/M = f[M/\xi(W)]$, we find $\xi \propto |W - W_c|^{-\nu}$ with $\nu = 2.42 \pm 0.01$ (the inset of figure 3(b)), which is very close to the value obtained for the lowest Landau band. Similar direct transitions can also be induced by reducing the strength of the magnetic field α. Figure 4(a) shows the critical disorder W_c for $n = 1 \to 0$, $2 \to 0$, and $3 \to 0$ transitions as a function of α. We can see that W_c monotonically decreases with the magnetic field strength which is extrapolated to zero at zero magnetic field. In figure 4(b), we plot the critical energy E_c versus the magnetic field α at $W = 0.2$ and 0.8. It is found that the extended state energy E_c for each Landau band is linearly proportional to the magnetic field and moves towards the direction of lower energy as disorder strength increases, consistent with the phase diagram obtained in [6], which predicts direct transitions from higher Hall plateaus ($n > 2$) to the insulator.

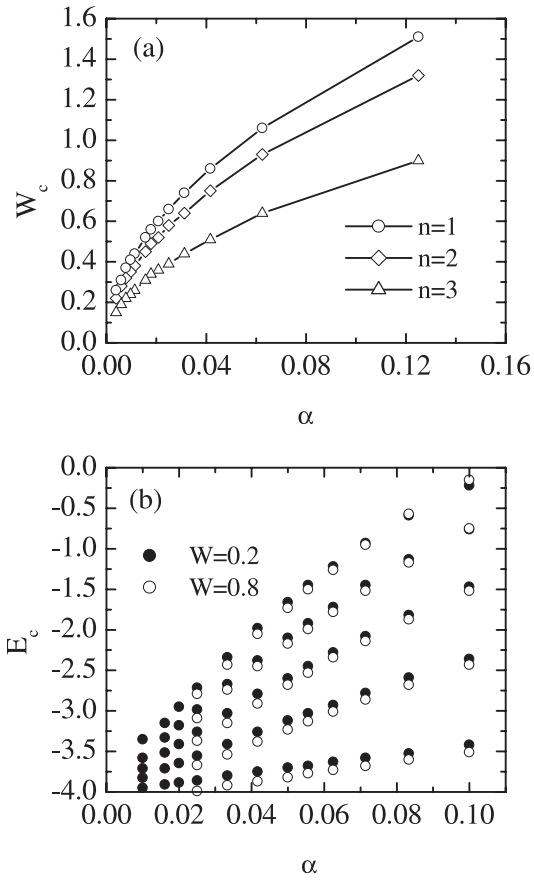

Figure 4. (a) The critical disorder W_c for $n = 1 \to 0$, $2 \to 0$, and $3 \to 0$ transitions as a function of α. (b) The critical energy E_c versus α at $W = 0.2$ and 0.8.

4. Summary

In summary, we have studied the disorder-driven quantum Hall state-to-insulator transition in a lattice model with only off-diagonal disorder using the well-developed transfer-matrix method. By increasing the strength of disorder, we found that the positions of the extended states do not float up in energy but move down in the direction towards lower energy. As a consequence, a direction transition from the IQHE to an insulator becomes possible. Our results are in agreement with the scenario proposed by Liu [6] and Sheng *et al* [8] based on the lattice model with diagonal disorder.

Acknowledgments

This work was supported by the National Natural Science Foundation of China. W S Liu is partially supported by the Natural Science Foundation of Henan Province of China.

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